

B.Sc. Part II (Hons) 4th Paper (Diff. Eqns.)
(LDECC (contd.))

Q Solve $(D^3 - 3D - 2)y = 540x^3 e^{-x}$.

Soln.

for CF

$$D^3 - 3D - 2 = 0$$

$$\Rightarrow D^3 + D^2 - D^2 - D - 2D - 2 = 0 \Rightarrow D^2(D+1) - D(D+1) - 2(D+1) = 0$$

$$\Rightarrow (D+1)(D^2 - D - 2) = 0 \Rightarrow (D+1)[D^2 - 2D + D - 2] = 0$$

$$\Rightarrow (D+1)[D(D-2) + 1(D-2)] = 0$$

$$\Rightarrow (D+1)(D-2)(D+1) = 0$$

$$\Rightarrow (D+1)^2(D-2) = 0 \Rightarrow D = -1, -1, 2.$$

$$\therefore \text{CF} = (C_1 + C_2 x)e^{-x} + C_3 e^{2x} \quad \text{--- (1)}$$

For PI

$$\text{PI} = \frac{1}{D^3 - 3D - 2} \cdot 540x^3 e^{-x}$$

$$\Rightarrow \text{PI} = \frac{540}{D^3 - 3D - 2} \cdot e^{-x} x^3 = \frac{540}{(D+1)^2(D-2)} \cdot e^{-x} x^3$$

$$\Rightarrow \text{PI} = 540 \cdot e^{-x} \frac{1}{[(D+1)^2][D-2]} x^3$$

$$\Rightarrow P_5 = 540 e^{-x} \frac{1}{D^2(D-3)} \quad x^3 = 540 e^{-x} \frac{1}{3D\left(\frac{D}{3}-1\right)} x^3$$

$$\Rightarrow P_5 = 540 e^{-x} \frac{1}{-3D^2\left(1-\frac{D}{3}\right)} x^3$$

$$\Rightarrow P_5 = 540 e^{-x} \frac{1}{-3D^2} \left(1-\frac{D}{3}\right)^{-1} x^3$$

$$\Rightarrow P_5 = -180 e^{-x} \frac{1}{D^2} \left(1 + \frac{D}{3} + \frac{D^2}{9} + \frac{D^3}{27} + \dots\right) x^3$$

$$\Rightarrow P_5 = -180 e^{-x} \frac{1}{D^2} \left[x^3 + \frac{D(x^3)}{3} + \frac{D^2(x^3)}{9} + \frac{D^3(x^3)}{27} + 0 \right]$$

$$\Rightarrow P_5 = -180 e^{-x} \frac{1}{D^2} \left[x^3 + \frac{3x^2}{3} + \frac{8x}{9} + \frac{6}{27} \right]$$

$$\Rightarrow P_5 = -180 e^{-x} \frac{1}{D^2} \left[x^3 + x^2 + \frac{2x}{3} + \frac{2}{9} \right]$$

$$\Rightarrow P_5 = -180 e^{-x} \frac{1}{D} \int \left(x^3 + x^2 + \frac{2x}{3} + \frac{2}{9} \right) dx$$

$$\Rightarrow P_5 = -180 e^{-x} \frac{1}{D} \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{2}{3} \frac{x^2}{2} + \frac{2}{9} x \right]$$

$$\Rightarrow P_5 = -180 e^{-x} \left[\frac{x^5}{20} + \frac{x^4}{12} + \frac{x^3}{9} + \frac{x^2}{9} \right] \quad (2)$$

Hence, the complete solution is given by $y = CF + PI$
 where CF and PI are given by (1) and (2) respectively.